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NEW RESULTS IN ONE-LOOP QUANTUM COSMOLOGY

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Abstract. A crucial problem in quantum cosmology is a careful analysis of the one-loop semiclassical approximation for the wave function of the universe, after an appropriate choice of mixed boundary conditions. The results for Euclidean quantum gravity in four dimensions are here presented, when linear covariant gauges are implemented by means of the Faddeev-Popov formalism. On using ζ-function regularization and a mode-by-mode analysis, one finds a result for the one-loop divergence which agrees with the Schwinger-DeWitt method only after taking into account the non-trivial effect of gauge and ghost modes. For the gravitational field, however, the geometric form of heat-kernel asymptotics with boundary conditions involving tangential derivatives of metric perturbations is still unknown. Moreover, boundary effects are found to be responsible for the lack of one-loop finiteness of simple supergravity, when only one bounding three-surface occurs. This work raises deep interpretative issues about the admissible backgrounds and about quantization techniques in quantum cosmology.

A careful consideration of boundary conditions is known to play a key role in several branches of classical and quantum field theory: the problems of electrostatics, the theory of vibrating membranes, the Casimir effect, the theory of van der Waals forces, models of quark confinement, the path-integral approach to quantum gravity and the quantum state of the universe. Indeed, a rigorous definition of the Feynman sum over all Riemannian four-geometries with their topologies does not yet exist. However, the choice of boundary conditions for metric perturbations and ghost modes may lead to a well defined elliptic boundary-value problem, and this may be applied to the one-loop semiclassical analysis of the quantum theory. Hence one may obtain a thorough understanding of the first set of quantum corrections to the underlying classical theory, which is a highly non-trivial achievement (despite the well known lack of perturbative renormalizability of Einstein's gravity). Such investigation leads, in turn, to a better understanding of mixed boundary conditions and of the effective action formalism in quantum field theory. Further motivations result from the quantization of closed cosmological models, and from the need to understand the relation between different approaches to quantum field theories in the presence of boundaries (e.g. reduction to physical degrees of freedom before quantization, or the Faddeev-Popov gauge-averaging method, or the extended-phase-space Hamiltonian path integral of Batalin, Fradkin and Vilkovisky). At this stage, one can indeed anticipate a striking result: in a mode-by-mode evaluation of the one-loop wave function of the universe, including gauge-averaging and ghost modes, after performing a 3+1 split and a Hodge decomposition of the components of metric and ghost perturbations, there are no exact cancellations between contributions of gauge and ghost modes. This lack of cancellation turns out to be essential to obtain agreement between different techniques [1].

We have first studied Luckock-Moss-Poletti boundary conditions on metric perturbations $h_{\mu\nu}$ and ghost perturbations φ_μ (hereafter, $\mu,\nu = 0,1,2,3$, K is the extrinsic-curvature tensor, g is the background metric, n is the normal to the boundary, and $P_{\mu\nu} \equiv g_{\mu\nu} - n_\mu n_\nu$):

$$[h_{ij}]_{\partial M} = [h_{0i}]_{\partial M} = 0 \, , \tag{1}$$

$$\left[(2\mathrm{Tr}K + n^\sigma \nabla_\sigma) n^\mu n^\nu (h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} h_{\alpha\beta}) \right]_{\partial M} = 0 \, , \tag{2}$$

$$[\varphi_0]_{\partial M} = 0 \, , \tag{3}$$

$$\left[(-K_\mu^{\ \nu} + \delta_\mu^{\ \nu} n^\rho \nabla_\rho) P_\nu^{\ \sigma} \varphi_\sigma \right]_{\partial M} = 0 \, . \tag{4}$$

These boundary conditions involve a mixture of Dirichlet and Robin boundary conditions, but unfortunately are not completely invariant under infinitesimal diffeomorphisms on metric perturbations. For this purpose, one has instead to consider the Barvinsky scheme:

$$[h_{ij}]_{\partial M} = 0 \; , \tag{5}$$

$$[\Phi_0(h)]_{\partial M} = [\Phi_i(h)]_{\partial M} = 0 \; , \tag{6}$$

$$[\varphi_0]_{\partial M} = [\varphi_i]_{\partial M} = 0 \; , \tag{7}$$

where Φ_0 and Φ_i are the normal and tangential components of the gauge-averaging functional. In linear covariant gauges, e.g. de Donder, Eqs. (6) lead to normal and tangential derivatives of h_{00} and h_{0i} [1].

Last, we have considered Robin-like boundary conditions on h_{ij}:

$$\left[\frac{\partial h_{ij}}{\partial \tau} + \frac{u}{\tau} h_{ij}\right]_{\partial M} = 0 \; , \tag{8}$$

jointly with

$$[h_{00}]_{\partial M} = [h_{0i}]_{\partial M} = 0 \; , \tag{9}$$

$$\left[\frac{\partial \varphi_0}{\partial \tau} + \frac{(u+1)}{\tau} \varphi_0\right]_{\partial M} = 0 \; , \tag{10}$$

$$\left[\frac{\partial \varphi_i}{\partial \tau} + \frac{u}{\tau} \varphi_i\right]_{\partial M} = 0 \; . \tag{11}$$

The resulting one-loop divergences on a portion of flat Euclidean four-space bounded by a three-sphere turn out to be, in the de Donder gauge [1],

$$\zeta(0) = -\frac{758}{45} \text{ (Eqs. (1) – (4))} \; , \tag{12}$$

$$\zeta(0) = -\frac{241}{90} \text{ (Eqs. (5) – (7))} \; , \tag{13}$$

$$\zeta(0) = \frac{89}{90} - u - 3u^2 + \frac{1}{3}u^3 \text{ (Eqs. (8) – (11))} \; . \tag{14}$$

By contrast, three-dimensional transverse-traceless (TT) perturbations yield $\zeta_{TT}(0) = -\frac{278}{45}$ in the cases (12) and (13), and $\zeta_{TT}(0) = \frac{112}{45} + 3u - u^2 - \frac{1}{3}u^3$ instead of the result in Eq. (14). It now remains to be seen what is the geometric form of heat-kernel coefficients when the Barvinsky boundary conditions (5)–(7) are imposed in the de Donder gauge. The

work in Ref. [2] seems to show that this leads to infinitely many new universal functions in Euclidean quantum gravity (such functions multiply all possible local invariants in a linear combination whose integration over ∂M yields the boundary part of heat-kernel coefficients).

If one wants to relate our analysis to the Lorentzian theory, one has also to interpret the quantum state of the universe corresponding to the boundary conditions studied so far in one-loop quantum cosmology. Moreover, the impossibility to restrict the measure of the Euclidean path integral to transverse-traceless perturbations raises further interpretative issues for the Lorentzian theory, where such a reduction to physical degrees of freedom is instead quite natural. Last, but not least, we find lack of one-loop finiteness of simple supergravity on manifolds with boundary, if one of the two boundary three-surfaces shrinks to a point [3]. Thus, many exciting open problems remain also in the understanding of boundary counterterms in supergravity theories.

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